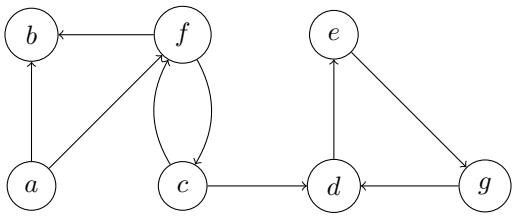
DSC 40B - Discussion 07

Problem 1.

Consider a *breadth*-first search on the graph shown in the figure, starting with node c.



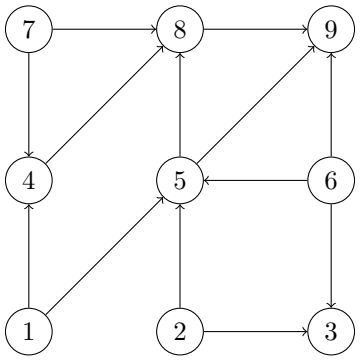
a) Suppose you call bfs_shortest_paths(graph, 'c') on the graph above. This function returns dictionaries distance and predecessor. Write down the contents of these dictionaries as they are when the function exits.

```
def bfs_shortest_paths(graph, source):
    status = {node:'undiscovered' for node in graph.nodes}
    distance= {node:float('inf') for node in graph.nodes}
    predecessor = {node: None for node in graph.nodes}
    status[source] = 'pending'
    distance[source]=0
    pending = deque([source])
    # while there are still pending nodes
    while pending:
        u = pending.popleft()
        for v in graph.neighbors(u):
            # explore edge (u,v)
            if status[v] == 'undiscovered':
                status[v]='pending'
                distance[v]=distance[u]+1
                predecessor[v]=u
                # append to right
                pending.append(v)
        status[u]='visited'
    return predecessor, distance
```

b) Mark the BFS trees produced on executing BFS on this graph.

Problem 2.

Consider the following directed graph.



a) Make a bold arrow from node u to node v if u is the predecessor of node v in DFS. Use the convention that a node's neighbors are processed in ascending order by label.

b) Fill in the table below so that it contains the start and finish times of each node after a DFS is performed on the above graph using node 1 as the source. Begin your start times with 1.

Node	Start	Finish
1		
2		
3		
4		
5		
6		
7		
8		
9		

c) Topologically sort the vertices of the graph.

Problem 3.

State whether the following statements are true or false.

a) Breadth first search on a directed graph always produces same number of BFS trees irrespective of order in which vertices are given and the neighbouring nodes are visited.

b) Breadth first search on an undirected graph always produces same number of BFS trees irrespective of order in which vertices are given and the neighbouring nodes are visited.

c) Both BFS and DFS require atleast $\Omega(V)$ memory.

d) Consider a graph G on which BFS is run with node s as the source. Assume that BFS visits a node u in the graph before node v. Then $d(s, u) < d(s, v)$

e) Every directed acyclic graph has exactly one topological ordering.

Problem 4.

Given an undirected graph G=(V,E), give an algorithm to find if the graph is disconnected.

Figure 1: "Full" DFS

```
from data classes import dataclass
    @dataclass
    class Times:
        clock: int
        start: dict
        finish: dict

    def full_dfs_times(graph):
        status = {node:'undiscovered' for node in graph.nodes}
        predecessor = {node: None for node in graph.nodes}
        times=Times(clock=0, start={}, finish={})
        for u in graph.nodes:
            if status[u]=='undiscovered':
                dfs_times(graph, u, status, times)
        return times, predecessor

        def dfs_times(graph, u, status, predecessor, times):
            times.clock+=1
            times.start[u]=times.clock
            status[u]='pending'
            for v in graph.neighbors(u):
                # explore edge (u, v)
                if status[v]=='undiscovered':
                    predecessor[v]= u
                    dfs_times(graph, v, status, times)
            status[u]='visited'
            times.clock+=1
            times.finish[u]=times.clock
```