DSC 40B

Theoretical Foundations II

Lecture 9 | Part 1

Warmup

Exercise

- How fast can we query/insert with these data structures?

	Query	Insert
Unsorted linked list	$\Theta(n)$	$\Theta(1)$
Unsorted array	$\Theta(n)$	$\Theta(n)$
Sorted array	$\Theta(\log n)$	$\Theta(\log n + n) = \Theta(n)$
BST	$\Theta(h)$	$\Theta(h)$
?	$\Theta(1)$	$\Theta(1)$

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Lecture 9 | Part 2

Direct Address Tables

Counting Frequencies

- How many times does each age appear?

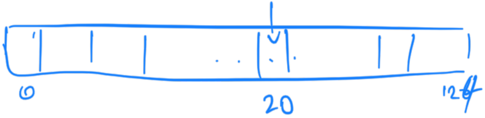
PID	Name	Age
A1843	Wan	24
A8293	Deveron	22
A9821	Vinod	41
A8172	Aleix	17
A2882	Kayden	4
A1829	Raghu	51
A9772	Cui	48
⋮	⋮	⋮

Exercise

What data structure would you use to store the age counts?

Direct Address Tables

- Idea: keep an **array** `arr` of length, say, 125.
- Initialize to zero.



- If we see age x, increment `arr[x]` by one.

Building the Table

```python
# loading the table
table = np.zeros(125)

for age in ages:
    table[age] += 1
```

- Time complexity if there are *n* people? $\Theta(n)$

Query

```
# query: how many people are 55?
print(table[55])
```

- Time complexity if there are *n* people? $\Theta(1)$

Counting Names

- How many times does each name appear?

PID	Name	Age
A1843	Wan	24
A8293	Deveron	22
A9821	Vinod	41
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⋮	⋮	⋮

Downsides

- DATs are **fast**.
- What are the downsides of DATs?
- Could we use a DAT to store:
  - zip codes?
  - phone numbers?
  - credit card numbers?
  - names?

Downsides

- Things being stored must be integers, or convertible to integers
  - why? valid array indices
- Must come from a small range of possibilities
  - why? memory usage. example: phone numbers

Hash Tables

- Insight: anything can be “converted” to an integer through **hashing**.
- But not uniquely!
- Hash tables have many of the same advantages as DATs, but work more generally.

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Lecture 9 | Part 3

Hashing

Hashing

- One of the most important ideas in CS.
- Tons of uses:
  - Verifying message integrity.
  - Fast queries on a large data set.
  - Identify if file has changed in version control.

Hash Function

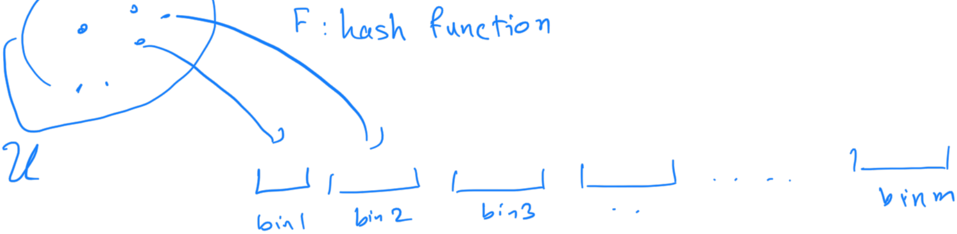
- A **hash function** takes a (large) object and returns a (smaller) "fingerprint" of that object.

- Usually the fingerprint is a number, guaranteed to be in some range.

$$F : \mathcal{U} \to \underbrace{[1, \ldots, m]}_{\text{range}}$$

How?

- Looking at certain bits, combining them in ways that *look* random (but aren't!)



Hash Function Properties

- Hashing same thing twice returns the same hash.
- Unlikely that different things have same fingerprint.
    - But not impossible!

Collisions

- Hash functions map objects to numbers in a defined range.
  - Example: given image, return number in $[0, 1, 2, \ldots, 1024]$
- There will be two images with the same hash.
  - **Pigeonhole principle**: if there are n pigeons, $< n$ holes, there will a hole with ≥ 2 pigeons.
- **Collision**: two objects have the same hash

“Good” Hash Functions

- A good hash function tries to minimize collisions.

Hashing in Python

- The `hash` function computes a hash.

```
>>> hash("This is a test")
-6704585799574772O3
>>> hash("This is a test")
-6704585799574772O3
>>> hash("This is a test!")
1860306055874153109
```

MD5

- MD5 is a **cryptographic** hash function.
  - Hard to “reverse engineer” input from hash.
- Returns a *really large* number in hex.

```
a741d8524a853cf83ca21eabf8cea190
```

- Used to “fingerprint” whole files.

Example

```
> echo ”My name is Justin” | md5
a741d8524a853cf83ca21eabf8cea190
> echo ”My name is Justin” | md5
a741d8524a853cf83ca21eabf8cea190
> echo ”My name is Justin!” | md5
f11eed2391bbd0a5a2355397c089fafd
```

Example

```
> md5 slides.pdf
e3fd4370fda30ceb978390004e07b9df
```

Why?

- I release a piece of software.
- I host it on Google Drive.
- Someone (Google, US Gov., etc.) decides to insert extra code into software to spy on users.
- You have no way of knowing.

Why?

- I release a piece of software & **publish the hash**.
- I host it on Google Drive.
- Someone inserts extra code.
- You download the software and hash it. If hash is different, you know the file has been changed!

Another Use: De-duplication

- Building a massive training set of images.
- Given a new image, is it already in my collection?
- Don’t need to compare images pixel-by-pixel!
- Instead, compare **hashes**.

Hashing for Data Scientists

- Don’t need to know much about *how* the hash function works.
- But should know how they are used.

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Lecture 9 | Part 4

Hash Tables

Membership Queries

- **Given**: a collection of n numbers and a target t.
- **Find**: determine if t is in the collection.

Goal

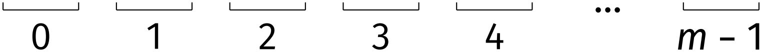
- DATs are fast, but won't work for things that aren't numbers in a small range.
- Idea: hash objects to numbers in a small range, use a DAT.
- But must deal with collisions.

Hash Tables

- Pick a table size m.
  - Usually $m \approx$ number of things you'll be storing.
- Create hash function to turn input into a number in $\{0, 1, \ldots, m - 1\}$.
- Create DAT with m bins.

Example

```
hash('hello') == 3
hash('data') == 0
hash('science') == 4
```

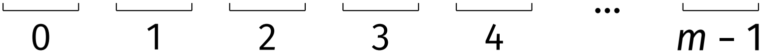


Collisions

- The **universe** is the set of all possible inputs.
- This is usually much larger than m (even infinite).
- Not possible to assign each input to a unique bin.
- If `hash(a) == hash(b)`, there is a **collision**.

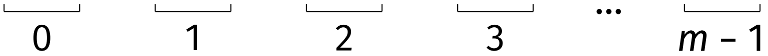
Example

```
hash('hello') == 3
hash('data') == 0
hash('san diego') == 3
```



Chaining

- Collisions stored in same bin, in linked list.
- **Query**: Hash to find bin, then linear search.



The Idea

- A good hash function will utilize all bins evenly.
  - Looks like uniform random distribution.
- If $m \approx n$, then only a few elements in each bin.
- As we add more elements, we need to add bins.

Average Case

- n elements in table.
- m bins.
- Assume elements placed randomly in bins[1].
- Expected bin size:

[1] Of course, they are placed deterministically.

Average Case

- n elements in table.
- m bins.
- Assume elements placed randomly in bins[1].
- Expected bin size: n/m

[1] Of course, they are placed deterministically.

Analysis

- Query:
  - Time to find correct bin:
  - Expected number of elements in the bin:
  - Time to perform linear search:
  - Total:

Analysis

- Query:
  - Time to find correct bin: $\Theta(1)$
  - Expected number of elements in the bin:
  - Time to perform linear search:
  - Total:

Analysis

- Query:
  - Time to find correct bin: $\Theta(1)$
  - Expected number of elements in the bin: n/m
  - Time to perform linear search:
  - Total:

Analysis

- Query:
  - Time to find correct bin: $\Theta(1)$
  - Expected number of elements in the bin: n/m
  - Time to perform linear search: $\Theta(n/m)$
  - Total:

Analysis

- Query:
  - Time to find correct bin: $\Theta(1)$
  - Expected number of elements in the bin: n/m
  - Time to perform linear search: $\Theta(n/m)$
  - Total: $\Theta(1 + n/m)$

Analysis

- Query:
  - Time to find correct bin: $\Theta(1)$
  - Expected number of elements in the bin: n/m
  - Time to perform linear search: $\Theta(n/m)$
  - Total: $\Theta(1 + n/m)$
  - We usually guarantee $m = O(n)$

Analysis

- Query:
  - Time to find correct bin: $\Theta(1)$
  - Expected number of elements in the bin: n/m
  - Time to perform linear search: $\Theta(n/m)$
  - Total: $\Theta(1 + n/m)$
  - We usually guarantee $m = O(n)$
  - Expected time: $\Theta(1)$.

Worst Case

- Everything hashed to same bin.
  - Really unlikely!
  - Adversarial attack?

- Query:
  - $\Theta(1)$ to find bin
  - $\Theta(n)$ for linear search.
  - Total: $\Theta(n)$.

Exercise

What is the worst case time complexity of inserting an element into a hash table that uses chaining with linked lists?

Growing the Hash Table

- Insertions take $\Theta(1)$ **unless** the hash table needs to grow.
- We need to ensure that $m \leq c \cdot n$.
  - Otherwise, too many collisions.
- If we add a bunch of elements, we'll need to increase m.
- Increasing m means allocating a new array, $\Theta(m) = \Theta(n)$ time.

Main Idea

Hash tables support constant (expected) time insertion and membership queries.

Dictionaries

- Hash tables can also be used to store (key, value) pairs.
- Often called **dictionaries** or **associative arrays**.

Hashing in Python

- `dict` and `set` implement hash tables.
- Querying is done using `in`:

```
>>> # make a set
>>> L = {3, 6, -2, 1, 7, 12}
>>> 4 in L # Theta(1)
False
>>> 7 in L # Theta(1)
True
```

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Lecture 9 | Part 5

Fast Algorithms with Hash Tables

Faster Algorithms

- Hashing is a super common trick.
- The “best” solution to interview problems often involves hashing.

Example 1: The Movie Problem

- You’re on a flight that will last D minutes.
- You want to pick two movies to watch.
- Find two whose durations sum to **exactly** D.

Recall: Previous Solutions

- Brute force: $\Theta(n^2)$.
- Sort, use sorted structure: $\Theta(n \log n) + \Theta(n)$.
- Theoretical lower bound: $\Omega(n)$?
- Can we speed this up with hash tables?

Idea

- To use hash tables, we want to frame problem as a **membership query**.

Example

- Suppose flight is 360 minutes long.
- Suppose first movie is fixed: 120 minutes.
- Is there a movie lasting (360 - 120) = 140 minutes?

```python
def optimize_entertainment_hash(times, D):
    hash_table = dict()
    for i, time in enumerate(times):
        hash_table[time] = i

    for i, time in enumerate(times):
        target = D - time
        if target in hash_table:
            return i, hash_table[target]
```

Example 2: Anagrams

Definition

Two strings `w_1` and `w_2` are **anagrams** if the letters of `w_1` one can be permuted to make `w_2`.

Examples

- `abcd` / `dbca`
- `listen` / `silent`
- `sandiego` / `doginsea`

Problem

- Given a collection of n strings, determine if any two of them are anagrams.

Exercise

Design an efficient algorithm for solving this problem. What is its time complexity?

Solution

- We need to turn this into a **membership query**.
- **Trick**: two strings are anagrams iff

```
sorted(w_1) == sorted(w_2)
```

```python
def any_anagrams(words):
    seen = set()
    for word in words:
        w = sorted(word)
        if w in seen
            return True
        else:
            seen.add(w)
```

Hashing Downsides

- Problem must involve **membership query**.

Example: The Movie Problem

- You're on a flight that will last D minutes.
- You want to pick two movies to watch.
- Find two whose added durations is **closest** to D.

Hashing Downsides

- No locality: similar items map to different bins.
- There is no way to quickly query entry closest to given input.

Example: Number of Elements

- Given a collection of n numbers and two endpoints, a and b, determine how many of the numbers are contained in $[a, b]$.

- Not a membership query.

- Idea: **sort** and use modified `binary search`.

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Lecture 9 | Part 6

Hash Table Drawbacks

Hashing **Downsides**

- No locality: similar items map to different bins.
- But we often want similar items at the same time.
- Results in many **cache misses**, **slow**.

Hashing Downsides

- Memory overhead.

Hash Tables vs. BSTs

- Hash Table: $\Theta(1)$ insertion, query (expected time).
- BST: $\Theta(\log n)$ insertion, query (if balanced).
- Why ever use a BST?

Hash Tables vs. BSTs

- Hash tables keep items in arbitrary order.
- Example: how many elements are in the interval $[3, 23]$?
- Example: what is the min/max/median?
- BSTs win when order is important.