DSC 40B - Homework 06

Due: Monday, November 13

Write your solutions to the following problems by either typing them up or handwriting them on another piece of paper. Unless otherwise noted by the problem's instructions, show your work or provide some justification for your answer. Homeworks are due via Gradescope at 11:59 p.m.

Problem 1.

In the following, let

$$V = \{0, 1, 2, 3, 4, 5, 6\},$$
$$E = \{(0, 2), (3, 2), (5, 6), (6, 1), (3, 0)\}.$$

For this problem, you do not need to show your work.

a) Draw the *undirected* graph $G = (V, E)$. Remember that when writing the edges of an undirected graph, we often abuse notation and write (u, v) when we really mean $\{u, v\}$; we have done so here.

b) Draw the graph $G = (V, E)$, assuming that G is directed.

c) Write down the connected components of G, assuming that G is undirected.

d) Write the adjacency matrix representation of G, assuming that G is undirected.

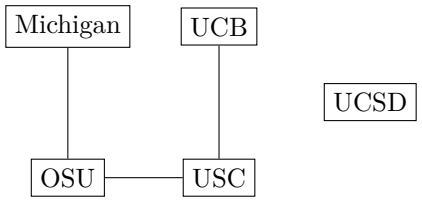
e) Write the adjacency matrix representation of G, assuming that G is directed.

Problem 2.

Suppose A is the adjacency matrix of an undirected graph. Let A^2 be the *squared* matrix, obtained by matrix multiplying A by itself. Show that the (i, j) entry of A^2 is the number of ways to get from i to j in exactly two hops (i.e., the number of paths of length two between node i and node j). *Hint:* Consult your linear algebra notes/textbook to remember a formula for the (i, j) entry of the product of two matrices.

Programming Problem 1.

We can use a graph to represent rivalries between universities. Each node in the graph is a university, and an edge exists between two nodes if those two schools are rivals. For instance, the graph below represents the fact that OSU and Michigan are rivals, OSU and USC are rivals, UCB and USC are rivals, but UCSD does not have a rival.



In a file called `assign_good_and_evil.py`, write a function `assign_good_and_evil(graph)` which determines if it is possible to label each university as either "good" or "evil" such that every rivalry is between a "good" school and an "evil" school. The input to the function will be a graph of type `UndirectedGraph` from the `dsc40graph` package. If there is a way to label each node as "good" and "evil" so that every rivaly

is between a "good" school and an "evil" school, your function should return it as a dictionary mapping each node to a string, `'good'` or `'evil'`; if such a labeling is not possible, your function should return `None`.

For example:

```
>>> example_graph = dsc40graph.UndirectedGraph()
>>> example_graph.add_edge('Michigan', 'OSU')
>>> example_graph.add_edge('USC', 'OSU')
>>> example_graph.add_edge('USC', 'UCB')
>>> example_graph.add_node('UCSD')
>>> assign_good_and_evil(example_graph)
{
    'OSU': 'good',
    'Michigan': 'evil',
    'USC': 'evil',
    'UCB': 'good',
    'UCSD': 'good'
}
```

If in the above graph, there is also an edge between `` `Michigan' `` and `` `USC' ``, then there does not exist such a label and your funtcion should return `None`.

Of course, there might be several ways to label the graph – your code need only return one labeling.