PID or Name: ______________________

DSC 40B - Sample Midterm 01

Note: This sample midterm is intended to give you an idea of the format of the exam, but it's not intended to be a comprehensive review of the material. Also, note that this sample exam is from a previous iteration of the course, and topics can change slightly from quarter to quarter depending on the instructor and how much was covered in lecture – you should expect Midterm 01 to cover the content from *this quarter*'s Lecture 01 through 08.

In addition to this sample exam, you should also study using the problems found at `https://dsc40b.com/practice`, as well as the labs and the homeworks.

Problem 1.

What is the time complexity of the following function? State your answer as a function of n using asymptotic notation in the simplest form possible. E.g., "$\Theta(n)$".

```
def foo(n):
    for i in range(n**3):
        for j in range(n):
            print(i + j)
        for j in range(n**2):
            print(i + j)
```

Problem 2.

What is the time complexity of the following function? State your answer as a function of n using asymptotic notation in the simplest form possible. E.g., "$\Theta(n)$".

```
def foo(n):
    for i in range(n):
        for j in range(i):
            for k in range(n):
                print(i + j + k)
```

Problem 3.

What is the time complexity of the following function? State your answer as a function of n using asymptotic notation in the simplest form possible. E.g., "$\Theta(n)$".

```
def foo(n):
    for i in range(200, n):
        for j in range(i, 2*i + n**2):
            print(i + j)
```

Problem 4.

What is the time complexity of the following function? State your answer as a function of n using asymptotic notation in the simplest form possible. E.g., "$\Theta(n)$".

```
import math

def foo(arr):
    """`arr` is an array with n elements."""
    n = len(arr)
    ix = 1
    s = 0

    while ix < n:
        s = s + arr[ix]
        ix = ix * 5 + 2

    return s
```

Problem 5.

The code below takes in an array of n numbers and checks whether there is a pair of numbers in the array which, when added together, equal the maximum element of the array.

What is the **best case** time complexity of this code as a function of n? State your answer using asymptotic notation.

```
def exists_pair_summing_to_max(arr):
    n = len(arr)
    maximum = max(arr)
    for i in range(n):
        for j in range(i + 1, n):
            if arr[i] + arr[j] == maximum:
                return True
    return False
```

Problem 6.

What is the **worst case** time complexity of the function in the last problem? State your answer using asymptotic notation.

Problem 7.

Consider again the problem of determining whether there exists a pair of numbers in an array which, when added together, equal the maximum number in the array. Additionally, **assume that the array is sorted**.

True or False: $\Theta(n)$ is a **tight** theoretical lower bound for this problem.

◯ True

◯ False

Problem 8. (2 points)

Suppose a and b are two numbers, with $a \leq b$. Consider the problem of counting the number of elements in an array which are between a and b; that is, the number of elements x such that $a \leq x \leq b$. You may assume for simplicity that both a and b are in the array, and there are no duplicates.

a) What is a **tight** theoretical lower bound for this problem, assuming that the array is **unsorted**? State your answer in asymptotic notation as a function of the number of elements in the array, n.

b) What is a **tight** theoretical lower bound for this problem, assuming that the array is **sorted**? State your answer in asymptotic notation as a function of the number of elements in the array, n.

Problem 9.

What is the **expected** time complexity of the following function? State your answer using asymptotic notation.

```
import random

def foo(n):
    x = random.randrange(n)

    if x < 8:
        for j in range(n**3):
            print(j)
    else:
        for j in range(n):
            print(j)
```

Problem 10.

What is the **expected** time complexity of the function below? State your answer using asymptotic notation. You may assume that `math.sqrt` and `math.log` take $\Theta(1)$ time. `math.log` computes the natural log.

```
import random
import math

def foo(n):
    # draw a number uniformly at random from 0, 1, 2, ..., n-1 in Theta(1)
    x = random.randrange(n)

    if x < math.log(n):
        for j in range(n**2):
            print(j)
    elif x < math.sqrt(n):
        print('Ok!')
    else:
        for i in range(n):
            print(i)
```

Problem 11.

State (but do not solve) the recurrence describing the runtime of the following function.

```
def foo(n):
    if n < 1:
        return 0

    for i in range(n**2):
        print("here")

    foo(n/2)
```

$$T(n) = \begin{cases} \Theta(1), & n = 1 \\ \boxed{}, & n > 1 \end{cases}$$

Problem 12. (2 points)

Solve the below recurrence, stating the solution in asymptotic notation. Show your work.

$$T(n) = \begin{cases} T(n/2) + \Theta(n) & n > 1 \\ \Theta(1) & n = 1 \end{cases}$$

Problem 13.

Suppose `bar` and `baz` are two functions. Suppose `bar`'s time complexity is $\Theta(n^3)$, while `baz`'s time complexity is $\Theta(n^2)$.

Suppose `foo` is defined as below:

```
def foo(n):
    if n < 1_000:
        bar(n)
    else:
        baz(n)
```

What is the asymptotic time complexity of `foo`?

Problem 14.

Let

$$f(n) = 5n \log n + \frac{n^3 + 5}{n + 2 + |\sin \pi n|} + n\sqrt{n}$$

Write f in asymptotic notation in as simplest terms possible.

$f(n) = \Theta($ $\square$ $)$

Problem 15.

Suppose $f_1(n)$ is $O(n^2)$ and $\Omega(n)$. Also suppose that $f_2(n) = \Theta(n^2)$.

Consider the function $f(n) = f_1(n) + f_2(n)$. True or false: it must be the case that $f(n) = \Theta(n^2)$.

- ◯ True
- ◯ False

Problem 16.

Suppose $f_1(n)$ is $O(n^2)$ and $\Omega(n)$. Also suppose that $f_2(n) = \Theta(n^2)$.

Consider the function $g(n) = f_2(n)/f_1(n)$. True or false: it must be the case that $g(n) = \Omega(n)$.

- ◯ True
- ◯ False

Problem 17.

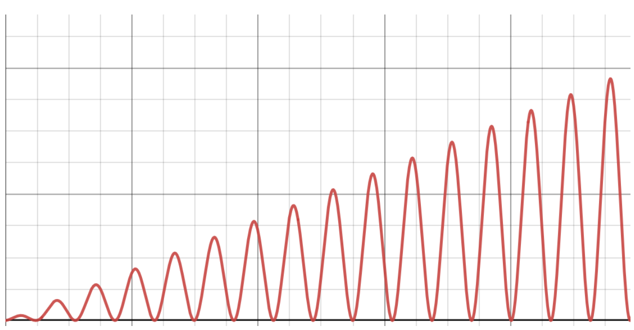
Suppose $f_1(n) = \Omega(g_1(n))$ and $f_2 = \Omega(g_2(n))$. Define $f(n) = \min\{f_1(n), f_2(n)\}$ and $g(n) = \min\{g_1(n), g_2(n)\}$.

True or false: it is necessarily the case that $f(n) = \Omega(g(n))$.

- ◯ True
- ◯ False

Problem 18.

Consider the function $f(n) = n \times (\sin(n) + 1)$. A plot of this function is shown below:



True or False: this function is $\Theta(n)$.

- ◯ True
- ◯ False

Problem 19.

Consider again the function $f(n) = n \times (\sin(n) + 1)$ from the previous problem.

True or False: $f(n) = O(n^3)$.

◯ True

◯ False

Problem 20.

Consider the iterative implementation of binary search shown below:

```
import math

def iterative_binary_search(arr, target):

    start = 0
    stop = len(arr)

    while (stop - start) > 0:
        print(arr[start])
        middle = math.floor((start + stop) / 2)
        if arr[middle] == target:
            return middle
        elif arr[middle] > target:
            stop = middle
        else:
            start = middle + 1
```

Which of the following loop invariants is true, assuming that `arr` is sorted and non-empty, and `target` is **not** in the array? Select all that apply.

☐ After each iteration, `stop - start >= 0`.

☐ After each iteration, `stop - start >= 1`.

☐ After each iteration, `arr[start] <= target`.

☐ After each iteration, `arr[start] <= max(target, arr[0])`.

Problem 21.

Consider `iterative_binary_search` from above and note the `print` statement in the `while`-loop. Suppose `iterative_binary_search` is run on the array:

```
[-202, -201, -200, -50, -20, -10, -4, -3, 0, 1, 3, 5, 6, 7, 9, 10, 12, 15, 22]
```

with target `11`.

What will be the last value of `arr[start]` printed?

☐

Problem 22.

Consider the code below which claims to compute the most common element in the array, returning a pair: the element along with the number of times it appears.

```python
import math

def most_common(arr, start, stop):
    """Attempts to compute the most common element in arr[start:stop]."""
    if stop - start == 1:
        return (arr[start], 1)

    middle = math.floor((start + stop) / 2)

    left_value, left_count = most_common(arr, start, middle)
    right_value, right_count = most_common(arr, middle, stop)

    if left_count > right_count:
        return (left_value, left_count)
    else:
        return (right_value, right_count)
```

You may assume that the function is always called on a non-empty array, and with `start = 0` and `stop = len(arr)`. Will this code always return the correct answer (the most common element)?

- ◯ Yes: it will always return the correct answer.
- ◯ No: it may recurse infinitely.
- ◯ No: it may try to access the array at an invalid index.
- ◯ No: it will run without error, but the element returned may not be the most common element in the array.

Problem 23.

Consider the modification of `mergesort` shown below, where one of the recursive calls has been replaced by an in-place version of `selection_sort`. Recall that `selection_sort` takes $\Theta(n^2)$ time.

```python
def kinda_mergesort(arr):
    """Sort array in-place."""
    if len(arr) > 1:
        middle = math.floor(len(arr) / 2)
        left = arr[:middle]
        right = arr[middle:]
        mergesort(left)
        selection_sort(right)
        merge(left, right, arr)
```

What is the time complexity of `kinda_mergesort`?

Problem 24.

Recall the partition operation from `quickselect`. Which of the following arrays could have been partitioned at least once? Select all that apply.

- ☐ `[50, 10, 20, 30, 60, 40]`
- ☐ `[20, 10, 30, 60, 40, 50]`
- ☐ `[20, 50, 10, 40, 30, 60]`
- ☐ `[60, 50, 40, 30, 20, 10]`
- ☐ `[10, 20, 30, 40, 50, 60]`

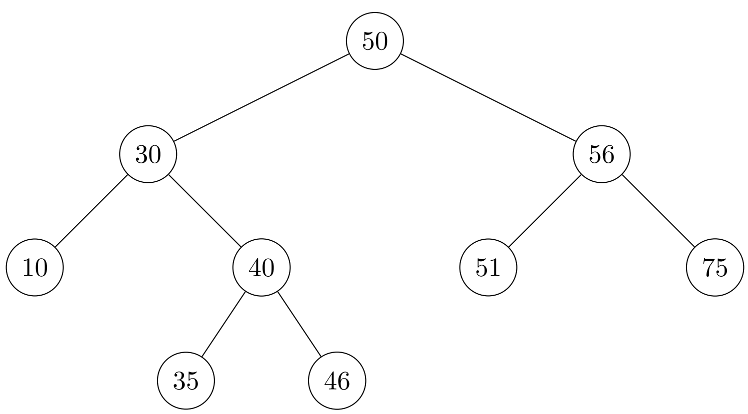
Problem 25.

Define the **largest gap** in a collection of numbers to be the largest difference between two distinct elements in the collection (in absolute value). For example, the largest gap in $\{4, 9, 1, 6\}$ is 8 (between 1 and 9).

Suppose a collection of n numbers is stored in a **balanced** binary search tree. What is the time complexity required for an efficient algorithm to calculate the largest gap of the numbers in the BST? State your answer as a function of n in asymptotic notation.

Problem 26.

Suppose the numbers `41`, `32`, and `33` are inserted (in that order) into the below binary search tree. Draw where the new nodes will appear.



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